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VIBRATIONS OF AN ANNULAR ISOTROPIC PLATE WITH ONE EDGE CLAMPED OR SIMPLY SUPPORTED AND AN INTERMEDIATE CONCENTRIC CIRCULAR SUPPORT

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1. INTRODUCTION

The present study deals with the determination of the exact fundamental frequency coefficient of the systems shown in Figure 1. Apparently no exact solution, as of this date, has been found to the problems under consideration [1].

2. SOLUTION OF THE PROBLEM

In the case of the systems shown in Figure 1 the exact solution of the problem governed by the classical partial differential equation is given by [1]

$$W_1(r) = A_i J_0(kr) + B_i Y_0(kr) + C_i I_0(kr) + D_i K_0(kr) \quad (i = 1, 2), \tag{1}$$

where $W_1(r)$ is the amplitude of axisymmetric normal modes of vibration for $a \leqslant r \leqslant c$ and $W_2(r)$ is the amplitude corresponding to the plate subdomain $c \leqslant r \leqslant b$.

The compatibility and boundary conditions are, in the case of the structural systems depicted in Figures 1(a) and 1(b):

$$W_1(c) = W_2(c) = 0, \qquad \frac{\mathrm{d}W_1}{\mathrm{d}r}(c) = \frac{\mathrm{d}W_2}{\mathrm{d}r}(c) \qquad \frac{\mathrm{d}^2 W_1}{\mathrm{d}r^2}(c) = \frac{\mathrm{d}^2 W_2}{\mathrm{d}r^2}(c), \tag{2–5}$$

$$\left.\frac{\mathrm{d}^2 W_1}{\mathrm{d}r^2} + \frac{\mu}{r}\frac{\mathrm{d}W_1}{\mathrm{d}r}\right|_{r=a} = 0, \qquad \left.\frac{\mathrm{d}}{\mathrm{d}r}\left(\frac{\mathrm{d}^2 W_1}{\mathrm{d}r^2} + \frac{1}{r}\frac{\mathrm{d}W_1}{\mathrm{d}r}\right)\right|_{r=a} = 0. \tag{6, 7}$$

When the plate is clamped at $r = b$, one has

$$W_2(b) = 0, \qquad \frac{\mathrm{d}W_2}{\mathrm{d}r}(b) = 0, \tag{8a, b}$$

and when it simply supported at $r = b$ the conditions are

$$W_2(b) = 0, \qquad \left.\frac{\mathrm{d}^2 W_2}{\mathrm{d}r^2} + \frac{\mu}{r}\frac{\mathrm{d}W}{\mathrm{d}r}\right|_{r=b} = 0. \tag{8c, d}$$

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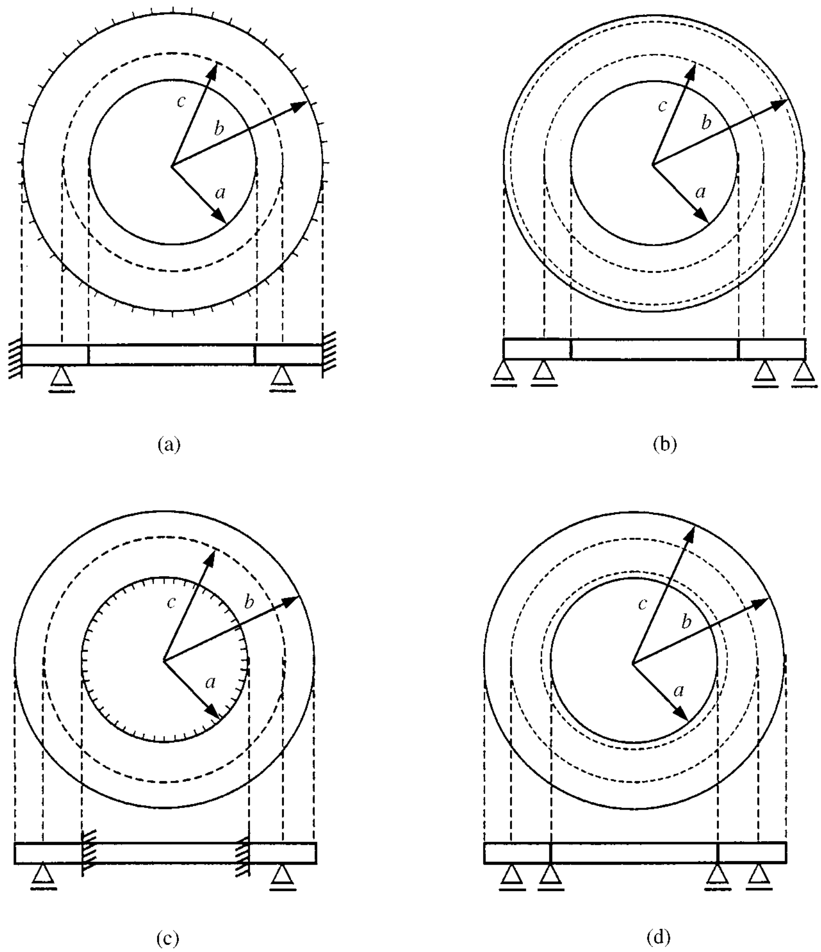

Figure 1. Vibrating structural systems considered in the present study: (a) Case I, (b) Case II, (c) Case III and (d) Case IV.

Substituting equations (1) into equations (2)–(8), one obtains a homogeneous system of equations in the A_i's, B_i's, C_i's and D_i's . The non-triviality condition yields, finally, an (8 × 8) determinantal equation in $kb = \sqrt[4]{(\rho h/D)}\,\sqrt{\omega}\, b$. Its lowest root is the square root of the fundamental frequency coefficient ($\Omega_1^{1/2}$). Accordingly $\Omega_1 = \sqrt{(\rho h/D)}\,\omega_i b^2$.

The procedure is the same in the case of systems (c) and (d) shown in Figure 1 (one substitutes "a" by "b" in equations (6) and (7) and "b" by "a" in equations (8)).

3. NUMERICAL RESULTS

All calculations have been peformed making the Poisson ratio (μ) equal to 0·3. They have been greatly facilitated by the use of MAPLE [2].

Tables 1–4 show values of Ω_1 for the mechanical arrangements depicted in Figures 1(a)–(d) respectively.

It is important to point out that the relative accuracy of the overall methodology has been tested considering some limiting cases by approaching the inner, circular support

TABLE 1

Fundamental frequency coefficients for an annular plate: with intermediate support, free inside and clamped outside (Case I)

a/b \ c/b	0·2	0·3	0·4	0·5	0·6	0·7	0·8	0·9
0·1	28·4626	35·3607	39·2159	30·6585	22·8710	17·7683	14·3376	11·9251
0·2		34·7037	42·6524	35·9611	25·5161	19·1019	15·0601	12·3404
0·3			45·2080	50·4234	34·2629	23·5876	17·5950	13·8962
0·4				62·6115	56·6659	34·2945	23·2462	17·2600
0·5					92·4077	62·0031	35·6415	23·9569
0·6						141·9110	68·1769	38·3823
0·7							200·1489	77·2059
0·8								255·9297

TABLE 2

Fundamental frequency coefficients for an annular plate: with intermediate support, free inside and simply supported outside (Case II)

a/b \ c/b	0·2	0·3	0·4	0·5	0·6	0·7	0·8	0·9
0·1	18·2397	22·9411	28·3512	27·6610	21·6856	17·0925	13·9442	11·7446
0·2		22·0313	28·5230	30·7249	23·8921	18·2738	14·6057	12·1398
0·3			28·6648	36·6636	31·0973	22·3167	16·9641	13·6344
0·4				39·9777	46·4116	31·8522	22·2164	16·8657
0·5					60·2784	55·0557	33·6329	23·2732
0·6						99·2180	62·9331	36·9690
0·7							165·8104	73·3350
0·8								235·5189

TABLE 3

Fundamental frequency coefficients for an annular plate: with intermediate support, clamped inside and free outside (Case III)

a/b \ c/b	0·2	0·3	0·4	0·5	0·6	0·7	0·8	0·9
0·1	4·9065	5·8746	7·2990	9·4915	13·0840	19·1558	25·1750	22·4073
0·2		6·2382	7·7815	10·1549	14·0843	21·1231	31·2105	29·3626
0·3			8·3435	10·9269	15·2241	23·1807	37·8763	39·9373
0·4				11·8575	16·5809	25·4717	44·5023	56·7685
0·5					18·2783	28·2266	51·1124	84·8294
0·6						31·8210	58·5482	129·9709
0·7							68·4728	183·9394
0·8								240·0768

towards the free edge of the plate. Excellent agreement with the eigenvalues available in reference [1] has been obtained. When the concentric support approaches the clamped or hinged inner plate support, the agreement with the values of reference [1] is also very good but for this situation the lowest frequency coefficient of the structural system does not correspond to an axisymmetric mode.

TABLE 4

Fundamental frequency coefficients for an annular plate: with intermediate support, free outside and simply supported inside (Case IV)

a/b \ c/b	0·2	0·3	0·4	0·5	0·6	0·7	0·8	0·9
0·1	4·8369	5·7196	7·0331	9·0617	12·3463	17·4989	20·8714	18·0514
0·2		6·1242	7·5103	9·6477	13·1563	19·0770	24·7558	21·5640
0·3			8·1591	10·4642	14·2844	21·1277	30·7563	28·1055
0·4				11·5430	15·7381	23·5217	37·9973	39·1970
0·5					17·6903	26·4796	45·6086	58·7941
0·6						30·5435	53·8599	95·1926
0·7							64·8495	154·4880
0·8								220·6059

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